Returns to scale at the University of Athens

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Abstract

Estimation of cost functions offers quantitative information for existing relations between total expenses and the level of output, while the theme of economies of scale is important for government decision making in many public owned activities, including that of education. This paper provides a framework for the measurement of economies of scale at the University of Athens, which is the most representative institution of the Greek Highest Educational System. Making use of annual data and a variation of cost functions and variables, we evaluate the importance of the $k$ -coefficient, which measures economies of scale. The basic conclusion of this case study is that the Institution is in the area of constant returns, a fact that must be seriously taken into account by all involved policy makers [JEL I].

Keywords : Costs, educational economics.

1. Introduction

The economics of education has been developed rapidly since the 1960’s and certainly has a long life to live in the future.

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In recent years, the role of returns to scale in education has been increasingly attracting the interest of both economists and educationalists. Many important current public policy issues in education can be greatly illuminated from the standpoint of returns to scale, offering useful insights today as in the past.

This study seeks to estimate function coefficients, (elasticities of returns-to-scale) for the University of Athens. This Institution is chosen as our unit of measurement, because it is the oldest, the largest, and the most historical University of the country, and hence representative of the whole Highest Educational System. As might be expected, the results and policy implications of this study can be useful to other Greek Institutions. This is particular true since the University of Athens, like the University of Salonica, is the only institution which is really a university in the classical sense as it is used all over the world, encompassing all sciences and arts. Finally, data availability was, also, a reason for our choice.

There are, various methods to estimate economies of scale, which, in general, are classified as the "economic approach" and the "engineering approach". If we follow the former, we have to examine the conditions of production in an effort to estimate the presence (or absence) of economies of scale. Alternatively, this may be accomplished by a simulating process of cost. Most of the studies in education have adopted a statistical approach of either cost or production functions.

This study estimates cost functions, despite of the (additional) difficulties involved in this approach, at least, according to our opinion. Furthermore, the study is at the University level, and no attempt is made to determine returns on individual facilities and departments. Time series data are used covering the period between 1975 and 1996, and the selected econometric technique is the OLS.

2. Educational production, cost and returns to scale: theory and practice

2.1 Underlying theory

If the historical dynamic process of the Greek educational system at the tertiary level could be observed from a macroscopic point of view, economies and/or diseconomies of scale as well as different public policies would be noticed. A monopoly or oligopoly structure could be also detected. There are legal and economic reasons for such a structure, although the social cost advantage of having a regulated "monopoly"
in highest education is a crucial element which seems to have been challenged by potential entrants, some economists and some politicians.

Apart from the debate, the University of Athens, as any other institution, combines inputs to "produce" educational services. Despite important differences existing between private firms and an education institution, the general idea remains the same. A university transforms factors of production into outputs.

The production set, $T$, is the set of all points $(x, y)$, where $x$ is a vector of inputs, $y$ is a scalar of output, such that $y$ can be produced from $x$.

(1) $(x, y) \in T$ or $xTy$ where $T \subseteq R^n \times R$ such that $xTy$ "asserts the possibility of the transformation of $x$ into $y$". Given this notation, returns to scale can be defined from a production approach.

(2) $V(x, y) \in T$, $(Ax, Ay) \in T, 1 < A < \infty$, $r > 1$ - (increasing) returns to scale, that is, the proportional increase in any input vector of a point on the production set can produce a greater than proportional increase in output.

An alternative way of looking at this matter is by using the production function, $y = F(x)$, $y \in R$, $x \in R^n$, which is the equivalent to the upper bound of $T$, i.e. $F(x) = \sup T$. Increasing returns to scale is then defined as

(3) $y = F(x) \Rightarrow A^r \cdot y = F(Ax), A > 1, r > 1$

or using the function coefficient (: scale elasticity)$^5$.

The transformation of inputs into outputs involves expenses for the University as for any other producing unit, in terms of both money (: factor inputs have their prices) and opportunity cost (: the cost of their best alternative use). The total cost of production is given by

$$c = \sum_{i=1}^{n} p_i \cdot i = c(y)$$

where $c(-)$ is the long-run total cost function, $y$ is a scalar of the level of output, and $p_i.s$ are the input prices$^4$.

There are increasing returns to scale (: economies of scale) if one of the following holds:

(1) $V(y, c)$, $c = c(y)$, $(dc)/(dy) > 0$, $(d^2c)/(dy^2) < 0$, that is, if total
costs increase at a decreasing rate.

(2) \( \sqrt{y(c)} \), \( \frac{c}{c} = \frac{d(c/y)}{(dy)} < 0 \), or elaborating the derivative,

\[
\frac{d}{dy} \left( \frac{c}{y} \right) = \frac{1}{dy} \left( \frac{d}{dy} \frac{c}{y} \right) < 0 \text{ that is, when marginal costs are lower than average total costs.}
\]

(3) \( k < 1 \) where \( k \) is the elasticity of total cost, — • -. It is essential to remember here that economies of scale (: \( k < 1 \)) implies increasing returns to scale (: \( E > 1 \)) since it is known in economics that \( k = \frac{1}{t} \).

(4) The elasticity of average cost, \( y \), is negative, where \( y = \frac{y}{c} = \frac{1}{e - 1} \), for \( 0 < k < 1 \), or \( y = \frac{1}{e} - 1 \), for \( e > 1 \).

2.2 Practical issues

a. The theoretical views cited above is an empty set if one does not specify the objective, the expenses occurred by the University overtime, the output "produced" and the cost function(s) to be used.

The "physical" conditions of offering educational services, the price of inputs and the supposed efficient conduct of a university determine the expenses occurred by the institution. This study is primarily interested in calculating the cost of the University of Athens over time, and it recognizes that economists are principally inclined towards the social cost of production, that is, the amount of other services that must be sacrificed by the State in order to use resources in education rather than in these other services. For instance, the educational resources might have been directed to Defence or the Health System. Opportunity costs must be, also, taken into account in any cost analysis. However, in empirical studies, researchers use either accounting or economic cost to estimate total expenses, depending on availability of data.

Researchers also discriminate between current and capital cost. The first includes all expenses concerning services and materials of immediate and short-run use, which are yearly renewed, while the latter includes expenses on capital goods with long-run returns.

Apart from the problem of how close measured expenditures represent cost as defined in theory, \(^5\) researchers have to decide between current and capital cost. In practice, both have been used depending, of course, on the objective of the research. In this study, total expenses are used for estimation, not only because it is compatible with the general
economic theory, but also because our topic concerns economies of scale over a long period of time. According to our stand, spreading overheads across many units of output is not the only reason for (fixed) cost per unit to decline. Thus, as the size of the University of Athens becomes larger over time, certain well-known forces work together to generate economies of scale (specialization and division of labor, technological factors), and/or diseconomies of scale later (limitations to efficient management). Hence, the present study considers both overheads and variable costs.

The method of forming the data herein is related to the type and source from which they originate. Basically this study uses data from the Ministry of Education and the National Statistical Service. The following items are included in current cost: expenditures on personnel (teaching staff, secretarial, auxiliary personnel), expenditures on stationery, bell and postal expenses, cost on books, transport expenses, and other expenses. Capital expenditures comprise items like buildings, machines, technical machinery and equipment, and new teaching facilities.

In all models, total cost is in terms of constant purchasing power, b. While the form in which the data are available often dictates the choice of the unit of the research (: university, school, department) and the type of analysis (: cross-section, time series), our main interest in the location of returns to scale over time for the University of Athens as a whole is attributed to other reasons as well. There has been a societal coercion on governments during the period between 1975 and 1996 to increase the number of students in Greek universities. This is mainly due to ethics, psychology and, in general, the "eccentric" temperament of the average Greek household, which is shaped by the particular conditions prevailing in Greek society and politics. The concomitant well-known debate between the State, pushing upwards the number of the students to enroll, and the University principles striving to restrain numbers, have ended up almost always with the State as winner for political reasons. Hence, since Greek universities are in essence owned and operated by the State, doubts have been cast upon the issue of whether the University of Athens has any further limits for increased enrollments. In other words, our hypothesis of investigation is whether economies of scale have existed during the sample period (: 1975-1997), and/or if they are approaching unity. If the latter is true, we believe a signal of alert must be sounded concerning the conditions which are necessary to minimize costs in the future. In conclusion, the University of Athens has been selected as
our unit of analysis, and cost functions will provide an answer to the relationship between its expenses and its size.

c. The choice of the independent variable(s) is the next subject to consider here. Researchers use a variety of measures, such as number of students on the roll\(^6\), teachers salaries, change in number of students, the average number of courses taught by teachers\(^7\), research activity, the student/staff ratio\(^9\), the rate of population growth and other demographic trends\(^10\), price trends in educational expenses\(^11\), and others. The key is, perhaps, to use as many regressors as possible to improve the fit. However, it is not clear, from the relevant empirical work, which regressors are the best so that a particular set might be selected on a priori grounds, - with the exception of the general concept of "output" measured with some method or index.

While certainly some regressors appear to improve the fit, others contribute either marginally, or not at all. The results are rather variant to the specification of the cost function, the unit of measurement\(^\wedge\) high school, university, etc), and the country concerned. While the inclusion of too many regressors at times creates statistical problems, the question of data availability is, also, a crucial factor in any analysis.

In the present study, five regressors are used as determinants of total expenses: \(n\) (the number of students in enrollment), \(ni\) (the number of postgraduates), \(Si\) (the student-staff ratio), \(nS\) (the proportion of postgraduates in the student body), and \(n^*\) (a compound index of students which is explained in the next section).

d. The choice of the functional form is the next issue to be decided upon. Many cost functions are available in economic theory and even more in empirical work, including the educational sector. The quadratic and cubic cost functions could not be excluded from our analysis, since it is known that both can define a minimum point on the average cost curve. Two more specifications are also used: the linear and the log-linear models. All models are combined with our regressors mentioned above, and standard regression and economic analysis is applied to estimate the parameters and the coefficients of economies of scale.

### 3. Empirical analysis

#### 3.1 Procedure

Two are our basic hypotheses with respect to the general procedure followed in the present study:
(a) The dependence of total expenses on output, or scale, which is represented by a cost function

\[ C = f(n, y) \]

where \( n \) denotes a measure of scale, or output, and \( y \) a vector of other variables, and

(b) Positive or negative economies of scale would cause total cost to rise less or more rapidly than output. Translated in terms of elasticity, this study seeks to estimate the coefficient of total cost elasticity, \( k \), as defined in section 2.2.

Four general conventional models were used to capture this effect:

a. The Linear Model
b. The C-D relationship
c. The Quadratic Model, and
d. The Cubic Function

For each model, various forms are estimated on the basis of the definition of output and the vector of additional variables. In particular, "output" is defined, alternatively, as \( n \) (the number of students on the roll) and \( M4 \) (a compound index for students). The construction of the index is made possible, since undergraduate students and postgraduates are almost non-joint "products" in the case of the University of Athens. The structure of graduate studies in this particular Institution, or, for this matter, in any other highest educational school in Greece, reinforces the fact that the two "products" are produced within the University using to a great degree separate production functions, in which certain inputs are specialized to each output. It follows that independent activities are carried on under the same roof, which is a non-genuine joint production process. This would, also, allow for the estimation of the proportion of the two outputs. This rationale is further supported by a priori considerations, since the departmental and university policy was to enroll a rather fixed number of graduate students as a function of the undergraduate students on the roll during the period between 1975 and 1996.

To the extent that these arguments are valid - pending the econometric analysis - we were able, on the one hand, to define a compound unit of output as \( k \) units of \( n \) (undergraduate students) and
1 unit of $T^2$ (postgraduate students) and to readjust the series of output to get 714, and, on the other hand, to experiment simultaneously with both regressors $n$ and $T^2$ in the Cobb-Douglas specification.$^1$

Standard regression analysis and econometric testing is applied for each model and its variants. In particular, selection among various specifications is based on the Akaike information criterion, while a restrictions test is run on the estimated scale coefficients. In addition, where is needed, a redundant or omitted variables test is conducted to test the statistical significance of a subset of our included variables.

In the presence of auto-correlation, equations are re-specified before using them for hypothesis tests, with the inclusion of auto-regressive (AR) and/or moving average terms. Serial correlation is detected by using the Breusch-Godfrey multiplier test for general high-order ARMA errors.

Heteroscedasticity may also create a problem, since in its presence, the conventional computed standard errors are no longer valid, even if the estimates are consistent. Detection is left to White's test, and, if present, correction of standard errors is made with the use of White consistent covariances estimator.$^1$

3.2 Results

Table 1 presents the estimated models. For each, only the selected specifications are shown. Not all models did well. Besides, all specifications did not have the same explanatory power or the same robust results. In general, all linear and log-linear specifications have invariably greater explanatory power than the quadratic model, while the cubic version gives very poor results. While effort was made for further selection, running the relevant tests as described in the last section (including a $t$-test for non-nested models)$^1$ we keep all forms in Table 1, not only because it is necessary to have a more spherical picture of overall results, but also, because all estimated forms, - with the exception of the cubic model - give robust effects concerning the economies of scale coefficient (see description below, and Table 1).

a. The Linear Model

From the many specifications of this form, four are kept for a brief review here. The selection is made as described in section 3.1 above. Regressor $n$, (the ratio of postgraduates to the student body) is dropped because its coefficient is not significantly different from zero. The only
exception is the one in which it is proved to be redundant, according to the relevant test. On the contrary, Sj (: the staff-student ratio) proves to work well, improving the fit when combined with \( n^\text{c} \) (: our compound index of student enrollment), while the results are poor when combined with \( n^\text{l} \) (: number on the roll). All four equations are free of serial correlation and heteroscedasticity. Both versions (i.e. with \( n^\text{l} \), and with \( n^\text{c} \)), prove to have relatively large explanatory power (: \( R^2 = 0.62 \) when \( n \) is the regressor, and \( R^2 = 0.67 \) when \( n^\text{c} \) is used). A \( t \)-test on these two specifications showed that the fitted values from specification I4 do not enter significantly in specification I3, but the fitted values of J4 do enter significantly in I5. Thus, we accept the specification with the \( n^\text{c} \) against that specified with \( n \).

Since the main hypothesis of interest is the scale elasticity, we estimate \( k \) for each equation. This can be seen from the table, using the slope of the fitted cost functions (: regression coefficients)\(^\text{17}\). Economies of scale are present in three equations, since elasticities are lower than unity, ranging from 0.843 to 0.857. The fourth elasticity is slightly greater than unity (: 1.071) in the estimation with no constant. Since it was felt on a priori grounds that the University of Athens is temporally and rapidly expanded due to the well-known social and political pressures to enroll more students than those approved by the Institution (: see section 2.2 above), we impose the restriction of constant returns to scale. In other words, we estimate a statistic measuring how close the unrestricted estimate comes to satisfying the restriction under the null hypothesis. In all four cases, the unrestricted estimates are close to satisfying the restriction. Therefore, the hypothesis of constant returns to scale is accepted (: See Table 1, column labelled \( HQ (: k = 1) \)).

b. The Log-linear Model

Table 1 presents further the results for the best versions of the Cobb-Douglas specification. Again, U3 is dropped from all specifications, because its coefficient is statistically insignificant, while Si proves to be an omitted variable by the relevant test, improving the fit in equations II1 and II3 which make use of only one output (: \( n^\text{l} \) and \( n^\text{c} \) alternatively). However, when Si is added to the two output version (: regressors log Hi and log 7*2), it renders very poor results since the coefficients of all variables turned out to be statistically equal to zero, and hence it is dropped from the equation. In fact, this version of Cobb-Douglas (: without S) shows that regressor log\( a \) is not statistically significant.
Once again, our prime interest is in the $k$ coefficients. One attractive feature of the double-log model is that the estimated coefficient of output measures directly the elasticity, of total cost, $k^{18}$. Even in this model, all computed elasticities are lower than one, within a small range from 0.830 to 0.975. The test of (the null hypothesis of) constant returns to scale is accepted in all versions.

c. The Cubic Model

All versions of the cubic model have invariably poor results. The ratios $S_i$ and $n^2$ are dropped, because their coefficients are not statistically significant and the relevant test give them as redundant regressors, either individually or together, when combined with both $n_1$ and $n_4$, alternatively. Regressors to the 1st, 2nd, and 3rd power (: for both $n_1$ and $n_4$) have, in general, the expected sign, but their coefficients are always insignificant at the presence of a statistically significant $R^2$. This indicates that this functional form is not appropriate. A $t$-test to choose between the two versions (: one containing $n_1$ and the other $n_4$) showed that neither specification is accepted (: see Model III, column labelled $j$-test). For the sake of an exercise, we calculated the average cost curve dividing both sides of the estimated total cost function by $n_1$, and we provided the first order condition for minimum average cost. This occurs with an enrollment of $n_1 = 42372$. Marginal costs are minimized at $n_1 = 14124$. According to this specification, optimal size has not been yet reached at the University of Athens. In fact this Institution is still on the falling portion of the AC curve, in contrast to the results given by the other models. But once again, no confidence at all can be placed on this no-linear form of total cost function.

d. The Quadratic Model

Turing to the quadratic model, we observe that only the equation with $n_1$ and with no additional variables give acceptable results (: selected model IV, Table 1). The student-staff and the graduate-student ratios are redundant, while the form with $n^2$, does very poorly with or without the ratios.

The equation is free of serial correlation and heteroscedasticity. Although this equation does not improve the fit compared to the rest models, it is used for an estimate of the average cost curve, and the
minimization process located economies of scale in the university of Athens, up to an enrollment of 30367 students, with the mean value of \( n \) (: number of students for the whole period to be about 32000 students). According to this formulation, the Institution has been already entering the flattened section of the average cost curve. This, is an indication that economies of scale are being exhausted. Even this model, then, reassures that we are in the area of constant returns to scale.

4. Conclusions

This study is the first attempt, as far as we know, to estimate product-specific economies of scale in the Greek Highest Educational System-either as whole or in specific institutions - using time-series analysis.

Alternative cost functions and regressors were used to shape a framework for the measurement of economies of scale coefficients. The policy dilemma, stated already in the paper, is obvious in the presence or absence of such economies.

It appeared initially that economies of scale were present at the University of Athens during the selected period, but the decisive relevant test on \( k \)-coefficients showed that economies of scale are being exhausted. This result suggests that the University of Athens has already reaped whatever benefits have existed from scale change. From now on, as the size increases over time, forces connected with inefficient management may take over and generate diseconomies of scale. Of course, if the institution is in the area of constant returns of scale, policy makers need to take it seriously into account, particularly by revising their practices to enroll additional students on political rather than on economic criteria.

Evidence for the optimum size cannot be extracted from most of our estimated models. But the minimization process in the quadratic model does show economies of scale up to 30367 students, a size somewhat smaller than the number of students the last years outside the sample period.

Finally, a caveat must be noted. This study is based on student enrollment, in various forms, to measure the size. Quality and other measures of size were not considered, primarily due to data limitations. Obviously, no interactions between scale and quality are allowed by our analysis, since overall efficiency is considered fixed. However, our overall goal was the location of economies of scale and/or the minimization of costs for the University of Athens, given the level of efficiency.
### Table 1
Estimated Models

<table>
<thead>
<tr>
<th>Model</th>
<th>Regressor</th>
<th>Coefficient</th>
<th>f-statistic</th>
<th>R²</th>
<th>Test on R²</th>
<th>d</th>
<th>F-test</th>
<th>H₀(βₑ=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Linear</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>i! C = flini</td>
<td></td>
<td>0.735</td>
<td>8.646</td>
<td>0.61</td>
<td>sign</td>
<td>1.6</td>
<td>Si: N₁ red</td>
<td>—</td>
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<tr>
<td>I₂ C = a₀ + fl₁o₄</td>
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<td>0.798</td>
<td>0.6</td>
<td>sign</td>
<td>1.62</td>
<td>N₁ red</td>
<td>— •</td>
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<tr>
<td>I₃ C = fl₁o₄ + fl₂S₁</td>
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<td>3.284</td>
<td>0.7</td>
<td>sign</td>
<td>1.81</td>
<td>—</td>
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<tr>
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<td>5649.2</td>
<td>0.796</td>
<td>0.6</td>
<td>sign</td>
<td>1.62</td>
<td>N₅S₁ red</td>
<td>—</td>
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<tr>
<td>II. Log-linear</td>
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<td></td>
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<tr>
<td>III Flgc = &quot;o + &quot;log₄t</td>
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<td>1.4610</td>
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<tr>
<td>log₄t</td>
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<td>2.616</td>
<td>Si: O₅</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
<td>IIII logC = fl₂log₄ + &quot;log₂S₁</td>
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<td>0.116</td>
<td>0.6</td>
<td>sign</td>
<td>1.99</td>
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<td>Accepted (against I₄)</td>
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<td>log₂S₁</td>
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<td>2.858</td>
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<tr>
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<td>—</td>
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<td>—</td>
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<tr>
<td>IIIIV logC = fl₀ + &quot;log₄n₄ + fl₂S₁</td>
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<td>-0.891</td>
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<td>sign</td>
<td>1.88</td>
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<td>log₄n₄</td>
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<td>-4.502</td>
<td>Si: O₅</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<tr>
<td>IIIIV logC = fl₀ + &quot;log₄n₄ + fl₂S₁</td>
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<td>1.6657</td>
<td>0.501</td>
<td>0.6</td>
<td>sign</td>
<td>1.74</td>
<td>log₂ &quot;n₄&quot;</td>
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<td>log₄n₄</td>
<td>0.8409</td>
<td>2.618</td>
<td>Si: N₅</td>
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<tr>
<td>IIIV logC = fl₀ + &quot;log₄n₄ + fl₂S₁</td>
<td>constant</td>
<td>-0.0338</td>
<td>-7.089</td>
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<td>log₄n₄</td>
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<td>-2.956</td>
<td>Si: N₅</td>
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(Contd. Table 1)
## Model Summary

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<tr>
<th>Model</th>
<th>Regressor</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>R²</th>
<th>Test on K²</th>
<th>d</th>
<th>R/0 test</th>
<th>/-test</th>
<th>k</th>
<th>$H_0 (: k = l)$</th>
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<td>III. Cubic</td>
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<td>sign</td>
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<td>N$_5$, S$_i$ : red</td>
<td>Rejected (: against III2)</td>
<td>$k &lt; F$</td>
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<td></td>
<td>-a</td>
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<td>-b</td>
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<td>-1.24</td>
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<td>+c</td>
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<td>1.204</td>
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<td>III$_2$</td>
<td>C = $\alpha_0 + \alpha_1 t + \alpha_2 s_1 + \alpha_3 s_2$</td>
<td>constant</td>
<td>-367620</td>
<td>-1.201</td>
<td>0.7</td>
<td>sign</td>
<td>1.15</td>
<td>N$_5$, S$_i$ : red</td>
<td>Rejected (: against IIIi)</td>
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<td>0.3633</td>
<td>1.232</td>
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<td>+c</td>
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<td>1.201</td>
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<td>IV. Quadratic</td>
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<td>constant</td>
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<td>-3.555</td>
<td>0.4</td>
<td>sign</td>
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<td>+b</td>
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**NOTES:**
- fe* minimum cost at 42372 students $A$:**
- minimum cost at 30261 students $d$:
- Durbin-Watson statistic sign: significant at the 5% level
- $R/0$ test redundant or omitted variables test(red = redundant, O = omitted) $K$
- economies of scale coefficient $H_0 ( : k = l )$: Null Hypothesis that $k = l$
- $s_1$: The compound index of student enrollment
- $N_5$: The proportion of postgraduates in the student body
- $s_2$: The number of postgraduates
- $S_i$: The staff-student ratio
Notes


4. It is assumed that the output-cost correspondence, \((y,c)\), is on the lower bound of the total cost set \((C > C(y))\), that is the institution chooses the optimal scale for the output level. This lower bound is the long-run total cost function, \(C = \inf C\).


8. See Layard and Verry (1975).


10. See Ta Ngoc (1972) and OECD (1976).


12. Returns to scale are easily extracted from the Knowledge of \(k\), given the inverse relation between total cost elasticity \(k\) and the function coefficient \(e\).

13. This is true up to 1995 at least. Lately, there have been certain changes and restrucure of the whole system of graduate studies.

14. If two products are produced in a fixed proportion \((n \mid f \mid n - k\), where \(k\) is a constant), the analysis for a single output can be applied in the place of the analysis of joint products.

15. White (1980).


17. For the linear model, the slope coefficient remains the same and the elasticity changes from point to point on the cost curve. In practice, the elasticity coefficient is usually computed at the sample mean values of the two variables to obtain the average elasticity.

18. In contrast with the linear model, the slope coefficient in the log-linear model is variable but the elasticity coefficient is constant.
References


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