A Semi-Analytical Model for Numerical Study of a Photonic Crystal Coupled Resonator Optical Waveguide with Disorder

Nikolaos Avaritsiotis, Thomas Kamalakis and Thomas Sphicopoulos

Abstract—This paper addresses the relation between fabrication imperfections and the performance of a photonic crystal Coupled Resonator Optical Waveguide (CROW). A semi-analytical model is presented, which calculates the perturbation of the coupling coefficients through their derivatives with respect to geometric characteristics of the rods of the photonic crystal lattice. Once these derivatives are calculated, it becomes possible to estimate the transfer functions of a large number of randomly perturbed devices at a small additional computational cost. This enables the statistical study of many performance issues of CROWs, such as the amplitude and position of the device's resonances. The model can be used to study the influence of imperfections of different strengths and types on CROWs of various lengths.

Index Terms—

I. INTRODUCTION

Coupled Resonator Optical Waveguides (CROWs) are currently attracting increased attention in view of their potential applications. A CROW constitutes a novel type of waveguide where light propagates through evanescent wave coupling through a large chain of identical optical resonators [1], [2]. Figure 1(a) illustrates a Photonic Crystal (PC) CROW formed by loosely coupled PC defect cavities. Two ordinary PC waveguides couple the light into and out of the cavities. Figure 1(b) and (c) illustrate an isolated PC cavity and waveguide respectively. Other resonator and waveguide types may be used; however PC-based structures offer the advantage of small dimensions and increased functionality [3]. From an application point of view, it is possible to realize sharp and reflection-less bends throughout the entire CROW band [1] by appropriately positioning the CROW resonators. CROWs with a small number of cavities can be used in optical filtering and modulation applications [4], [5]. CROWs may also be used to drastically slow down the optical field opening up a path towards the realization of integrated optical delay lines [6]. The low group velocity enhances the nonlinear behaviour of the device, making it attractive for all-optical signal processing as well [7].

Fabrication induced disorder [8] may influence the coupling properties of the resonators and the input/output waveguides, thereby altering several properties of the CROW. These issues have been addressed using perturbation theory [9]. In [10], quantitative relationships were given, which describe how the group velocity is affected. A dual harmonic modulation of the refractive index is proposed in [11] for 1D CROWs to ensure virtually identical resonator interaction. These studies focus on infinite CROWs and do not consider the coupling of light in and out of the device. In finite devices, this coupling determines the position and the spectral width of the CROW resonances.

To account for random perturbations in finite devices, Finite Difference (FD) schemes such as the FD Time Domain (FDTD) [12] and the FD Frequency Domain (FDFD) [13],[14] methods require very small grid size in order to capture small geometric perturbations. To obtain reliable statistical results, many perturbed devices must be calculated rendering such simulations intractable. In this paper, an alternative approach is proposed based on a previously developed coupled mode model [15]. The model is applied to the calculation of the derivatives of the coupling coefficients with respect to the rod radii and positions. Once these derivatives are calculated, one may estimate the transfer functions of a large number of devices with randomly perturbed geometric characteristics using Taylor’s expansion. A similar procedure was applied in [16] to study the effect of disorder in a PC coupler.

The rest of the paper is organized as follows: In section II, the general model used for estimation of the transfer function of an ideal CROW device is briefly presented. In section III, the calculation of the cavity/cavity and cavity/waveguide coupling coefficients is discussed and semi-analytical formulas are derived to facilitate the calculations. Section IV, discusses the calculation of the perturbations of the coupling coefficients through their derivatives. Section V summarizes the simulation procedure and in section VI, the model is used to study the influence of imperfections in the position and bandwidth of the CROW resonances, assuming different disorder levels. Some concluding remarks are given in section VII.
the effect of imperfections as well. \(S\) is a plane normal to the propagation of the waveguide modes \(z\). Assuming tight field confinement, only the coupling between the waveguides and the edge cavities needs to be considered. Hence \(k_{0,1}\(z\)), \(k_{0,1}\(z\))=0, for \(p>1\) and \(k_{2,1}\(z\))=0, \(k_{2,1}\(z\))=0 for \(p<N\) where \(N\) is the number of cavities. The cavity-cavity mode coupling is described by the elements of an \(N\times N\) matrix \(K=K_{pq}\) given by

\[
K_{pq} \equiv \iint (\omega e^{-\omega t}e_{\rho}) E_0^* E_0 dV
\]

(5)

It is possible to normalize the elements of \(K_{pq}\) so that \(K_{pq}=K_{pq}^*\). Note that \(K_{pq}\) are real quantities since the cavity electric field modes are also real. Due to the weak coupling one can assume that \(K_{pq}=0\) when \(p-q\geq 1\), i.e., that the matrix \(K\) is tri-diagonal.

We may also define the functions \(\Lambda_{\beta,\rho}(z)\) and \(\Lambda_{\beta,\eta}(z)\), related to the accumulated waveguide/cavity:

\[
\begin{align*}
\Lambda_{\beta,\rho}(z) & = \int_{z_1}^{z} k_{\beta,\rho} dz, \\
\Lambda_{\beta,\eta}(z) & = \int_{z_1}^{z} k_{\beta,\eta} dz
\end{align*}
\]

(6)

where \(z=z_1\) and \(z=z_2\) are the input and output of the device, respectively (see Figure 1(a)). Using these functions, the components of the vectors \(K^{\beta,\rho}_{pq}\) and \(K^{\beta,\eta}_{pq}\) can be defined as:

\[
K^{\beta,\rho}_{pq} = -j\Lambda^*_{\beta,\rho}(z_2), \quad K^{\beta,\eta}_{pq} = j\Lambda^*_{\beta,\eta}(z_2)
\]

(7)

In the case of weak coupling, \(K^{\beta,\rho}_{pq}(z)\approx 0\) and \(K^{\beta,\eta}_{pq}(z)\approx 0\), for \(q>1\) and \(K^{\beta,\rho}_{pq}(z)\approx 0\). \(K^{\beta,\eta}_{pq}(z)\approx 0\) for \(q\leq N\). An \(N\times N\) matrix \(J=J_{pq}\) may now be defined, where:

\[
J_{pq} = \sum_{i=1}^{N} (k_{\beta,\rho}^{\beta,\rho}_{pq} - k_{\beta,\rho}^{\beta,\rho}_{pq}) dz
\]

(8)

The element \(J_{pq}\) of \(J\), depends on the coupling of the \(p\)th and the \(q\)th CROW cavities to the input and output waveguides. Since the fields are tightly confined only \(J_{1}\) and \(J_{N}\) of \(J\) have non-negligible values. Table 1 shows the coupling coefficients calculated for an ideal (unperturbed) PC CROW using the Plane Wave Expansion (PWE) method as in [15]. The rod dielectric constant is \(\varepsilon_0=11.56\), while that of the background is \(\varepsilon_0=1\). The lattice constant is \(a=0.6\mu m\) while the rod radii is \(r_0=0.18a\). The cavity/cavity and waveguide/cavity spacings are denoted by \(w_1\) and \(w_2\) respectively. The table shows that both the real and imaginary parts of \(J_{1}\) and \(J_{N}\) are significantly smaller than \(K_{12}\) and that \(|Re\{J_{11}\}|\) is larger than \(|Im\{J_{11}\}|\).

Using integration by parts as in [17], it is easy to show that:

\[
Re\{J_{11}\} = -\frac{1}{2}|\Delta_{11}(z_1)|^2 - \frac{1}{2}|\Delta_{11}(z_2)|^2
\]

(9)
TABLE I  
CROW COUPLING COEFFICIENTS (IN Hz)  

| $\text{Re}(J_{11})$ | $\text{Im}(J_{11})$ | $|K_{12}|$ | $|\text{Re}(J_{11})|$ | $|\text{Im}(J_{11})|$ | $|K_{12}|$ |
|-----------------|-----------------|----------|-----------------|-----------------|----------|
| $3.3 \times 10^{11}$ | $2.8 \times 10^{10}$ | $6.1 \times 10^{12}$ | $2 \times 10^{10}$ | $4.6 \times 10^{9}$ | $3 \times 10^{3}$ |

B. Estimation of the Transfer Function

Using the matrices and vectors defined above and the fact that $K_{11}(0) = 0$ for $q > 1$ and $K_{11}(0) = 0$ for $q = N$, one may estimate the transfer function, of light exiting from waveguide 2 as [15]:

$$H(\Delta f) = \frac{a_{ij}(z_{i})e^{j\beta f}}{a_{ij}(z_{i})} = e^{j\beta f} p_{n,1} A_{n,i}^{(N)} A_{i,j}^{(N)} \tag{10}$$

where $p_{n,i}$ are the elements of $P = (J + jK)^{T}$, $\beta$ is the propagation constant of the waveguide modes and $\Delta f = \Delta \omega / 2\pi$. Since $(J + jK)P = I$, the element $p_{n,1}$ of the inverse can be calculated from the system of equations

$$(j2\pi f + J_{11})p_{11} + jK_{12} p_{21} = 1$$

$$(j2\pi f + J_{21})p_{12} + jK_{22} p_{22} + jK_{23} p_{31} = 0$$

$$(j2\pi f + J_{32})p_{22} + jK_{32} p_{23} + jK_{34} p_{41} = 0$$

$$\vdots$$

$$(j2\pi f + J_{N,i})p_{N-1,i} + jK_{N-1,i} p_{N-1,i+1} = 0$$

$$\vdots$$

$$(j2\pi f + J_{N,N})p_{N,i} + jK_{N-1,N} p_{N,i} = 0 \tag{11}$$

Using some algebraic manipulation, it is possible to show that if $\text{Im}(J_{11})$ is neglected, as suggested by Table 1, one obtains $p_{n,1}(-\Delta f) = p_{n,1}(\Delta f) \equiv |p_{n,1}(\Delta f)|$ and hence the transfer function obeys the following symmetry condition

$$H(\Delta f) = H(-\Delta f) \tag{12}$$

Equation (12) holds for both the unperturbed and perturbed devices (since no assumptions are made for $K_{11}$ and $J_{11}$ other than $|\text{Im}(J_{11})| \neq 0$). This symmetry condition will be useful in explaining the behavior of the perturbed transfer functions later in the text. Figure 2, illustrates the power transfer functions $T(f) = |H(f)|^{2}$ calculated using (10) assuming a PC CROW with 10 cavities for two rod ($w = 0.25\text{m}$) and three rod spacing ($w = 0.5\text{m}$). The resonance lobes of $T(f)$ are distributed symmetrically as expected from (12). As noted in [15], the number of resonances equals the number of cavities, and the resonances are sharper near the edges of the passband. It is therefore reasonable to expect that imperfections will mostly influence the edge resonances. The spacing between the resonances gets narrower, as the coupling is weakened. The peaks of the ideal transfer function $T(f_{0})$ are all at 0.25 [18], where $f_{0}$ are the resonance positions.

Figure 2: Ideal transfer functions for a CROW with 10 cavities for a) 2 rod cavity/cavity and waveguide/cavity spacing and b) three rod spacing.

C. The three cavity CROW

For a small number of cavities, it may be possible to further simplify (10) by carrying out the matrix inversion analytically. For a three cavity CROW and assuming that $J_{11} = K_{NN} = J$ where $J$ is purely real, one can obtain the transfer function as:

$$H(\Delta f) = \frac{A_{0}}{(j2\pi f - z_{1})} + \frac{A_{1}}{(j2\pi f - z_{2})} + \frac{A_{2}}{(j2\pi f - z_{3})} \tag{13}$$

where

$$A_{0} = -K_{12} K_{23} K_{13}^{(0)} / (K_{12}^{2} + K_{23}^{2}) \tag{14}$$

$$A_{1} = A_{0} z_{2} / (z_{2} - z_{1}) \tag{15}$$

and $z_{0} = -J$ while $z_{1}$, $z_{2}$ are the roots of $z^{3} + Jz + K_{12}^{2} + K_{23}^{2} = 0$. For large $K_{12}$ and $K_{23}$, it is easy to see that the roots $z_{1}$ and $z_{2}$ are almost purely imaginary, and

$$z_{1} \equiv z_{2} = -\frac{1}{2} J + j \left[ \sqrt{K_{12}^{2} + K_{23}^{2}} \right] = \frac{1}{2} z_{0} + j \left[ \frac{K_{12}^{2} + K_{23}^{2}}{2} \right] \tag{16}$$

One then obtains three sharp distinct resonances centered at $f = -\text{Im}(z_{1}) / 2\pi$ while the 3dB bandwidth is determined by $\text{Re} |z_{1}| / \pi$. The value of the transfer function $|H(\Delta f)|$ at the peaks is determined by $|A_{1} / \text{Re} |z_{1}| |$ and using (16), one may show that

$$T(f) = |H(\Delta f)|^{2} = \left| A_{1} / \text{Re} |J| \right|^{2} \tag{17}$$

Equation (17) holds even in the case where $K_{12}$ and $K_{23}$ are not equal. The 3 cavity model presented in this subsection may serve as a guide in order to better understand the results presented in the following sections.

III. ESTIMATION OF THE COUPLING COEFFICIENTS

In this section the model presented in [15] is extended with the intention to incorporate the effect of perturbations. In order to make the calculation easier, instead of calculating the matrices $J$ and $K$ and the vectors $K_{11}$ and $K_{22}$ from (3)-(8) using numerical integration, one may expand the field in plane waves and calculate the coupling coefficients in (3),(4) and (5) in a simpler procedure. For example, in a 2D PC structure with dielectric rods, the electromagnetic field of the modes of the cavities and waveguides is purely transverse [19], and one may express the electric field of the cavity modes $E_{d} = E_{d0} N$, where:
\[ E_{py} = \sum_n v_n \exp(jG_n r) \]  

(18)

and \( y \) is the direction of the rods. The vectors \( G_m \) are the reciprocal lattice vectors of the cavity supercell (Figure 1(b)), given by \((m_2\pi/w_2)x+(m_2\pi/w_2)z\), where \( m_1 \) and \( m_2 \) are integers and \( a_0 \) is chosen large enough so that the modal field is negligible at the supercell edge. Using (18) and (5) one obtains:

\[ K_{py} = \Delta \omega \sum_{G_m} v_{py}^* v_{py} \int \left( \varepsilon - \varepsilon_0 \right) \exp\left( \left[ G_m - G_n \right] \cdot r \right) dA \]  

(19)

where \( r=(x,0,z) \) and \( dA=dxdz \). The set of rods \( R_g = R_C \cup R_t \) that should be taken into account in (19), contains all of the rods of the CROW (set \( R_C \)) and the isolated cavity (set \( R_t \)). The radii and positions of the CROW rods \( R_C \) may be perturbed due to imperfections. One can therefore write the following equation for \( \varepsilon - \varepsilon_0 \):

\[ \varepsilon(r) - \varepsilon_0(r) = \left( \varepsilon - \varepsilon_0 \right) \sum_{v=\pm} \lambda_v S(r - r_v, R_v) \]  

(20)

where \( \lambda_v \) equals 1 if the \( v^\text{th} \) rod belongs to the CROW (\( \nu \in R_C \)) and -1 if it belongs to the isolated cavity (\( \nu \in R_t \)). The function \( S(r,R) \) is equal to 1 for \( |r| \leq R \) and equal to zero for \( |r| > R \). In (20) \( \varepsilon_0, \varepsilon_0 \) are the dielectric constants of the rods and the background respectively, \( r_v = (x_v,z_v) \) are the centers of the rods, \( R_v \) are their radii. The 2D Fourier integral of \( S(r-R) \) which can be easily calculated using [19]:

\[ F(G,R) = \frac{1}{2\pi} \int S(r-R) e^{iGR} dA = \begin{cases} \frac{2\pi R}{G} \text{J}_1(GR) & G \neq 0 \\ \frac{\pi R^2}{G} & G = 0 \end{cases} \]  

(21)

Using equations (19), (20) and (21) one obtains the following expression for \( K_{py} \):

\[ K_{py} = \Delta \omega \left( \varepsilon - \varepsilon_0 \right) \sum_{v=\pm} \sum_{G_m} v_{py}^* v_{py} e^{-iG_m \cdot r_v} \lambda_v F(G_m, R_v) \]  

(22)

In (22), \( G_m = G_x - G_y \), and \( G_m = G_y - G_x \). Equation (22) provides a means to calculate the cavity/cavity coupling coefficients from the planewave expansion of the cavity modes without any numerical integration. Imperfections can be directly incorporated by simply adding small perturbations \( \Delta x, \Delta z, \Delta R, \nu \) in the positions and radii of the CROW rods (set \( R_C \)).

The electric field of the waveguide modes may also be expanded in terms of plane waves. For example, the forward mode of the input waveguide is expanded as:

\[ E_{f_{1,y}} = e^{ij_0} \sum_n U_n \exp(jG_n r) \]  

(23)

In (23), the vectors \( G_n^* \) are the reciprocal lattice vectors of the waveguide supercell (see Figure 1(c)), given by \((m_2\pi/w_2)x+(m_2\pi/w_2)z\), where \( w_1 \) and \( w_2 \) are the sizes of the supercell in the \( x \) and \( z \) direction respectively. In the 2D case, the integrals in (3)-(4) are reduced in one dimension (along \( x \)). Substituting (18) and (23) in (3), and performing the integration, one obtains:

\[ k_{f_{1,1}}(z) = \Delta \omega \left( \varepsilon - \varepsilon_0 \right) \sum_{v=\pm} \sum_{G_m} \lambda_v e^{i(G_m \cdot r_v)} U_n^* U_n \varepsilon_0 (G_m - G_n, z) \]  

(24)

where the function \( \varepsilon_0 \) is zero when \( |z-z_v| > R_v \) and if \( |z-z_v| \leq R_v \), then \( \varepsilon_0 \) is chosen large enough so that the modal field is negligible at the supercell edge. Using (22) and (24) one obtains:

\[ g_v(G_v, z) = \left\{ \begin{array}{ll} e^{i(G_v \cdot z_0)} & G_v \neq 0 \\ 0 & G_v = 0 \end{array} \right. \]  

(25)

In (25), \( Z_0(z_1) \) and \( Z_0(z_2) \) are the intersection of the \( v \)-th rod circumference with the line \( z=z_1 \) (see Figure 3). Equations (22) and (24) can be used to calculate the coupling coefficients \( K_{pq} \) and \( k_{11}(z) \) and 1D numerical integration could then be used to obtain \( K_{pq} \) and \( K_{p_0} \) as well as \( J_{11} \) and \( J_{SN} \) according to the equations of the previous section. This is further discussed in the next section.

**IV. GEOMETRIC PERTURBATIONS**

If the perturbations are small, Taylor’s expansion up to first order may be used to speed up the computations. Assuming small rod displacements by \( \Delta x, \Delta z, \Delta R, \nu \) along \( x \) and \( z \) and a small change in their radii, \( \Delta R, \nu \), then \( K_{pq} \) may be expanded as:

\[ K_{pq} = K_{pq,0} + \sum_{v=\pm} \left\{ \frac{\partial K_{pq}}{\partial x_v} \Delta x_v + \frac{\partial K_{pq}}{\partial z_v} \Delta z_v + \frac{\partial K_{pq}}{\partial R_v} \Delta R \right\} \]  

(26)

where \( K_{pq,0} \) is the unperturbed value of \( K_{pq} \) and the derivatives in (26) are calculated assuming the rods are located at their ideal positions. The derivatives maybe accurately estimated using finite difference formulas. To estimate \( \frac{\partial K_{pq}}{\partial x_v} \), for example, one simply calculates the values \( K_{pq,1} \) and \( K_{pq,2} \) of \( K_{pq} \) assuming that all the rods are at their ideal positions, except rod \( \mu \) whose \( x \) position is perturbed by \( +h \) and \( -h \) respectively (for small \( h \)). The derivative is approximated as:

\[ \frac{\partial K_{pq}}{\partial x_v} \approx \frac{K_{pq,1} - K_{pq,2}}{2h} \]  

(27)

In (27), the coupling coefficients \( K_{pq,1} \) and \( K_{pq,2} \) are subtracted and hence one needs to consider only the terms of the sum in
According to (6), to calculate the coupling coefficients $K_{pq}$ with $p-q \leq 1$ need to be considered.

Furthermore, rods situated far away from the $p^{th}$ and $q^{th}$ cavity have negligible bearing on the value of $K_{pq}$. As illustrated in Figure 4, for the calculation of $K_{12}$ one needs to consider the rods that are close to the 1st and 2nd cavity. There are some symmetry considerations that may further reduce the number of derivative calculations that need to be performed. For example, if a change in the radius of rod “1” by $\Delta R_1$ produces a change in $\nu$ and $\mu$ of rod “2”, then changing the radius of rod “4” and “7” by $\Delta R_1$ will produce the same change in $K_{23}$ and $K_{14}$ respectively and so on. Similar symmetry considerations apply to the rest of the CROW rods as well.

For example, if a change in the radius of rod “1” by $\Delta R_1$ produces a change in $\nu$ and $\mu$ of rod “2”, then changing the radius of rod “4” and “7” by $\Delta R_1$ will produce the same change in $K_{23}$ and $K_{14}$ respectively and so on. Similar symmetry considerations apply to the rest of the CROW rods as well.

A similar expansion can also be applied to calculate the change in the coupling coefficients using Taylor’s expansion presented in this section. Hence in the calculations of $J_{NN}$, it is better to split the interval into three parts that are closer to the ideal rod while the solid line to the perturbed rod.

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To calculate the change $\Delta J_{11}$ in the real part of $J_{11}$ we use (9) to approximate $\Delta J_{11}$ as:

$$\Delta J_{11} = -\text{Re} \left\{ \Lambda_{11} (z_1) \Delta A_{11} (z_1) + \Lambda_{12} (z_2) \Delta A_{11} (z_2) \right\}$$  \hspace{1cm} (30)

In (30), the second order terms $|\Delta A_{12} (z_1)|^2$ and $|\Delta A_{11} (z_2)|^2$ are ignored. Similar approach is used for the estimation of the other derivatives of $J_{NN}$. Hence in the calculations of these derivatives, one can use $h=10^{-12}$m.

The estimation of the perturbations of the coupling coefficients using Taylor’s expansion presented in this section is the final ingredient of the model which is summarized in section V.

$$K_{12}$$

![Figure 4: Rods considered in the calculation of the cavity/cavity coupling coefficients.](image)

![Figure 5: The three types of perturbations $\Delta x$, $\Delta z$, and $\Delta R$ considered in the analysis and the subsections required for accurate numerical integration. The dashed lines correspond to the ideal rod while the solid line to the perturbed rod.](image)

![Figure 6: The estimated derivative of $\text{Re}\{J_{11}\}$ with respect to the displacement of rod “A” in Figure 4.](image)

**V. SUMMARY OF THE SIMULATION PROCEDURE**

In this section the procedure for the simulation of the perturbed CROW devices is briefly summarized. The first step is to calculate the coupling coefficients $K_{12}$, $K_{11}$, and $K_{2N}$ of the ideal device. This can be carried out using equations (19)
and (24). The matrix $K=[K_{pq}]$ is constructed setting $K_{pq}=K_{12}$ for $|p-q|=1$ and $K_{pq}=\Delta \alpha$, while in all other cases one sets $K_{pq}=0$. 1D numerical integration is then used to calculate the ideal values of $K^{(1)}_{f}$ and $K^{(2)}_{f}$ and the vectors $K_{f}=[K^{(1)}_{f} \ldots 0]^T$ and $K_{f2}=[0 \ldots K^{(2)}_{f} 0]^T$. The elements $J_{11}$ and $J_{N2}$ which are the only non negligible elements of $J$ are estimated from (8). Given the matrices $J$ and $K$ and the vectors $K_{f}$ and $K_{f2}$, equation (10) can be used to estimate the transfer function of the ideal device.

To incorporate the effect of perturbations, the derivatives of $K_{pq}$, $\Lambda_{1,1}(z_{1})$, and $\Lambda_{1,2,2}(z_{1})$ need to be calculated using the procedure outlined in the previous section. Then one generates random perturbations $\Delta x_{i}$, $\Delta r_{i}$, and $\Delta R_{i}$ for each rod of each perturbed device. Equations (26), (29) and (30) are used to obtain the perturbations of the matrices $J$ and $K$ and the vectors $K_{f}$ and $K_{f2}$. We note again, that the derivatives need to be calculated only once and can then be used to statistically study the effect of imperfections on a large number of perturbed devices.

VI. RESULTS AND DISCUSSION

The model presented in the previous sections will now be used in order to study the effect of imperfections the performance of the CROW. Figure 7, shows the transfer functions of 200 perturbed devices plotted on the same axis, corresponding to a 10 cavity CROW with a) 2 rod spacing ($w_{1}=w_{2}=3a$) and b) three rod spacing ($w_{1}=w_{2}=4a$). The perturbations are assumed Gaussian random variables with zero mean value and standard deviation equal to $\Delta x=<\Delta x>_{\sigma_{x}}=\frac{1}{2}\sigma_{x}$ and $\Delta R=<\Delta R>_{\sigma_{R}}=\frac{1}{2}\sigma_{R}$. As shown in the figure, imperfections cause a random displacement of the centers and a change in the amplitudes of the resonances. This is more pronounced for larger $w_{1}$ and $w_{2}$, since the ideal coupling coefficients become smaller and more susceptible to the influence of perturbations. It is also interesting to note that the effect of imperfections is more severe near the edge of the passband (as predicted in section II).

Figure 7: Transfer functions for perturbed CROWs with 10 cavities for a) 2 rod cavity/cavity and waveguide/cavity spacing and b) three rod spacing. The rod positions and radii are perturbed by 2nm ($\Delta=2nm$).

Table II and Table III, provide an estimate on the uniformity of the transfer function for a 5 cavity and a 10 cavity CROW respectively. For a given transfer function $T(f)$, one may define $\sigma$, as the standard deviation $\sigma=|\text{std}\{ T(f_{n}) \} |$, where $f_{n}$ are the resonance centers. A filter where all resonances have the same amplitude (i.e. $T(f_{n})=T(f_{0})$ and consequently $\sigma=0$) is more suitable for telecom applications. In tables II and III, the average $\sigma$ is calculated for 1000 CROW devices with either $w_{1}=w_{2}=3a$ or $w_{1}=w_{2}=4a$ and $\Delta=1nm$, 2nm and 3nm. It is deduced that $\sigma$ gets larger as $\Delta$ or $w_{1}$ and $w_{2}$ are increased implying a larger non-uniformity. For 3nm perturbations, $\sigma$ is 0.02 and 0.056 for the 5 and 10 cavity three spacing CROWs implying an average deviation around the ideal value (=0.25) of about 8% and 22% respectively. The influence of the device size is illustrated in Figure 8, where the values of $\sigma$, calculated from 200 sample devices each, is shown with respect to the number of CROW cavities $N$ for $w_{1}=w_{2}=3a$ and $\Delta=2nm$. The figure indicates that non-uniformity rapidly increases with the size of the device, implying that larger CROWs are much more susceptible to fabrication imperfections.

**TABLE II**

<p>|5 CAVITY CROW|
|---|---|---|---|---|---|
| &amp; $w_{1}=w_{2}=3a$ | $w_{1}=w_{2}=4a$ |</p>
<table>
<thead>
<tr>
<th>1nm</th>
<th>2nm</th>
<th>3nm</th>
<th>1nm</th>
<th>2nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>$1.4 \times 10^{-4}$</td>
<td>$5.4 \times 10^{-4}$</td>
<td>$1.3 \times 10^{-3}$</td>
<td>$2.7 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

**TABLE III**

<p>|10 CAVITY CROW|
|---|---|---|---|---|---|
| &amp; $w_{1}=w_{2}=3a$ | $w_{1}=w_{2}=4a$ |</p>
<table>
<thead>
<tr>
<th>1nm</th>
<th>2nm</th>
<th>3nm</th>
<th>1nm</th>
<th>2nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>$4.4 \times 10^{-4}$</td>
<td>$1.4 \times 10^{-3}$</td>
<td>$3 \times 10^{-3}$</td>
<td>$0.01$</td>
</tr>
</tbody>
</table>

Figure 9, presents an insight on the way the amplitude of the resonances are influenced by imperfections. The amplitudes $T(f_{1})$ and $T(f_{2})$ of the first ($m=1$) and middle ($m=N/2$) resonances obtained by 10 randomly perturbed devices are plotted in the case of a 4 and a 20 cavity CROW. The strength of the imperfections is $\Delta=2nm$. One notices that in both cases there exists a small deviation from the ideal value $T(f_{0})=0.25$. For the 4 cavity CROW there seems to be a perfect match...
between $T(f)$ and $T(f_{NC})$. As the cavity number is increased to 20, this match becomes worse, implying a larger non-uniformity. This match is rather difficult to explain in the general case. It is useful however to consider the 3 cavity CROW where according to (17), the transfer function will have the same value at all three peaks and this holds regardless of the perturbations of the coupling coefficients $K_{12}$ and $K_{23}$. It is also interesting to study the resonance frequency shift. Figure 10 presents the standard deviation $\sigma_m$ of the frequency displacement of the resonances $\sigma_m=\text{std}(f_m)$ for $w_1=w_2=3a$ and $w_1=w_2=4a$ for various values of the imperfection strength $\Delta$. As anticipated, the effect of imperfections is stronger at the edges of the passband. The figure suggests that the lobes of the transfer function are displaced symmetrically around the center of the CROW band ($f= f_0$ or $\Delta=0$). This is justified with the symmetry condition imposed by (12). The figure also suggests that in the case of an odd cavity CROW the central resonance does not shift due to the perturbations. This is again consistent with (12). For an odd number of cavities, the central resonance of the ideal (unperturbed) device will be near $\Delta=0$. In the perturbed device, the central lobe may not move to the right or left of $\Delta=0$ since this would break the symmetry condition of (12). This is why there is almost no frequency shift for the central resonance. This can also be explained considering the 3 cavity CROW model that was presented in Section II. The positions of the central resonance frequency is primarily determined by $\text{Im}[J]$ whose ideal value as shown in Table I is much smaller compared to the 3dB bandwidth of the resonance determined by $\text{Re}[J]$. Hence even if the perturbations of $\text{Im}[J]$ are considered, these should be negligible compared to the resonance bandwidth to cause any noticeable frequency shift. One may also notice that for a device with $w_1=w_2=4a$ the values of $\sigma_m$ appear to be smaller than those for the case of $w_1=w_2=3a$. However since the resonance bandwidth is much smaller when $w_1=w_2=4a$, the perturbations are eventually more influential.

![Figure 10: The normalized standard deviation $\sigma_m$ of the frequency locations of the resonances $f_m$ of the transfer function $T(f)$. Plots (a) and (c) correspond to a 5 cavity CROW device, whereas plots (b) and (d) to a 10 cavity CROW device. In addition (a) and (b) refer to $w_1=w_2=3a$ while (c) and (d) to $w_1=w_2=4a$. - corresponds to $\Delta=1nm$, - to $\Delta=2nm$ and - to $\Delta=3nm$. All values of Figure 10 appear normalized and in order to get a more practical view of the influence of imperfections on the resonance frequencies, Tables IV and V provide the std of the frequency displacement of the first resonance for several $\Delta$ values. Note that even for small perturbations ($\Delta=1nm$), the frequencies can be displaced by several GHz which could prove detrimental depending on the application.

<table>
<thead>
<tr>
<th>TABLE IV</th>
<th>5 CAVITY CROW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$ (MHz)</td>
<td>$\Delta=1nm$</td>
</tr>
<tr>
<td>$w_1=w_2=3a$</td>
<td>$2.8\times10^4$</td>
</tr>
<tr>
<td>$w_1=w_2=4a$</td>
<td>$2.4\times10^4$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE V</th>
<th>10 CAVITY CROW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$ (MHz)</td>
<td>$\Delta=1nm$</td>
</tr>
<tr>
<td>$w_1=w_2=3a$</td>
<td>$2.5\times10^4$</td>
</tr>
<tr>
<td>$w_1=w_2=4a$</td>
<td>$2.4\times10^4$</td>
</tr>
</tbody>
</table>

Figure 11 illustrates the influence of fabrication induced disorder on the 3dB bandwidth $\Delta f_{3dB}$ of each resonance. The standard deviation $\sigma_{f3dB}=\text{std}(\Delta f_{3dB})$ of $\Delta f_{3dB}$ for each resonance is plotted for several $\Delta$ values. One notices that the values of $\sigma_{f3dB}$ are smaller for resonances at the edges of the passband. Since these resonances are sharper however, even a small displacement can significantly alter the relative resonance bandwidth.
In this paper, the influence of fabrication-induced disorder in the performance of a photonic crystal CROW was numerically investigated. A semi-analytical model was proposed for the calculation of the transfer function of the device in the presence of geometric perturbations of various types. The model is an extension of a previous model [15] for the simulation of an ideal device, and is based on the estimation of the derivatives of the coupling coefficients of the CROW. Once these derivatives are calculated, a large number of perturbed devices may be simulated with a very small computational overhead. This enables the statistical study of various performance issues such as the amplitude change and frequency shift of the device resonances as well as the resonance 3dB bandwidth. The model presented in this paper may be used to numerically establish the relation between the device performance and the quality of the fabrication process.

VII. CONCLUSIONS

REFERENCES


Nikolaos Avaritsiotis (nickavasi@di.uoa.gr) obtained his BSc in Physics from the University of Thessaloniki in 2004 and MSc in analogue and digital integrated circuit design from Imperial College of London in 2005. He is currently working towards his PhD degree in the subject of photonic crystal devices.

Dr. Thomas Kamalakis (thkm@di.uoa.gr) obtained his BSc in Informatics, MSc in Telecommunication with distinction and PhD in the field of integrated optics, from the University of Athens in 1997, 1999 and 2004 respectively. Since 2008, he is a lecturer in the department of Informatics and Telematics in the Harokopion University of Athens. Dr. Kamalakis is the author and co-author of more than 50 publications in scientific journals and conferences in the fields of integrated optics, nanophotonics, optical fiber propagation and optical detection and optical wireless systems. He is a member of the Optical Society of America (OSA).

Thomas Spicopoulos (thom@di.uoa.gr) received the Physics degree from Athens University in 1976, the D.E.A. degree and Doctorate in Electronics both from the University of Paris VI in 1977 and 1980 respectively, the Doctorat Es Science from the Ecole Polytechnique Federale de Lausanne in 1986. From 1976 to 1977 he worked in Thomson CSF Central Research Laboratories on Microwave Oscillators. From 1977 to 1980 he was an Associate Researcher in Thomson CSF Aeronautics Infrastructure Division. In 1980 he joined the Electromagnetism Laboratory of the Ecole Polytechnique Federal de Lausanne where he carried out research on Applied Electromagnetism. Since 1987 he is with the Athens University engaged in research on Broadband Communications Systems. In 1990 he was elected as an Assistant Professor of Communications in the Department of Informatics & Telecommunications, in 1993 as Associate Professor and since 1998 he is a Professor in the same Department. His main scientific interests are Microwave and Optical Communication Systems and Networks and Techno-economics. He has lead about 40 National and European R&D projects. He has more than 100 publications in scientific journals and conference proceedings. From 1999 he is advisor in several organisations including EETT (Greek NRA for telecommunications) in the fields of market liberalisation, spectrum management techniques and technology convergence.